جامعة الملك عبدالله
للعلوم والتقنية
King Abdullah University of
Science and Technology

# Course Syllabus: Statistics of Extremes

## STAT 380 - Lecture STAT 380

| Course Information | |
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| **Comprehensive Course Description** | ***Description:*** This advanced statistics course aims at providing a deep understanding of Extreme-Value Theory results, models, and methods, as well as some experience in the practical application of these tools to real data using the statistical software R. Theoretical and practical aspects will be covered.<br>***Tentative week-by-week outline:***<br>-W1: **Introduction:** Motivation; Examples; Brief history of the field developments;<br>-W1-2: **Fluctuations of Sums:** Convergence; Laws of large numbers; Glivenko-Cantelli Theorem; Law of the iterated logarithm; Stable laws; Central Limit Theorem; Domains of attraction;<br>-W2: **Fluctuations of Maxima:** Max-stability; Extremal Types Theorem (ETT); Generalized Extreme-Value (GEV) distribution; Max-domains of attraction; Von Mises conditions;<br>-W3-4: **Statistical Analysis of Maxima:** Application of the ETT; Return levels; Moment and likelihood estimation; Bayesian extremes; MCMC; Metropolis-Hastings algorithm;<br>-W5-6: **Threshold Approaches:** Point processes; Limiting results; Peaks Over Threshold (POT) analysis; Generalized Pareto Distribution (GPD); r-largest statistics method;<br>-W7: **Modeling of Non-stationarity:** Non-stationarity; Inference and diagnostics; Semi-parametric regression; Quantile regression;<br>-W8-9: **Temporal Dependence:** Time series; Stationarity; D(un) and D(un) conditions; Clustering and cluster size distribution; Extremal index; Return levels; Declustering;<br>-W10-12: **Multivariate Extremes:** Componentwise maxima; Standardization; Limit distribution; Exponent measure; Pickands' dependence function; Parametric models; Multivariate point process; Multivariate GPD; Extrapolation; Asymptotic dependence and independence; Dependence measures; Likelihood inference; Composite likelihood Ledford-Tawn model and extensions; Heffernan-Tawn approach;<br>-W13-15: **Spatial Extremes:** Gaussian processes; Stationarity and isotropy; Correlation functions; Max-stable processes; Limiting results for pointwise maxima; de Haan's representation; Parametric models; Current challenges. |
| **Course Description from Program Guide** | This advanced statistics course aims at providing a rather deep understanding of Extreme-Value Theory results, models, and methods, as well as some experience in the practical application of these tools to real data using the statistical software R. Theoretical and practical aspects will be covered. Topics covered include (a) Univariate Extreme- Value Theory: Extremal-Types Theorem; GEV distribution; return levels; Domains of attraction; Threshold-based methods; GPD distribution; Point process representation; r-largest order statistics approach; Likelihood inference; Modelling of non-stationarity; Dependent time series; Clustering and declustering approaches. (b) Multivariate Extreme-Value Theory: Modelling of componentwise maxima; Spectral representation; Parametric models; Dependence measures; Asymptotic dependence/independence; Threshold methods; Likelihood-based inference. (c) Spatial Extremes: Gaussian processes; correlation functions; Max- stable processes and models. |
| **Goals and Objectives** | ***Objectives:*** At the end of this course, the students should be able to:<br>-Understand, state, prove all results seen in class and in the weekly assignments.<br>-Apply extreme-value statistics tools in real data applications, fit extreme-value models using various techniques, interpret the results and diagnostic plots, know how to use the statistical software R in this context, and understand and interpret R outputs.<br>-Master Extreme-Value Theory and Statistics in the univariate, multivariate, and spatio-temporal case, when extreme events are defined as block maxima or threshold exceedances. |
| **Required Knowledge** | **Prerequisites**: STAT 220, 230, 250 (Mandatory); AMCS 310, 370 (Recommended) |

<table>
<tr><td>Reference Texts</td><td>
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<li>Coles (2001) An Introduction to Statistical Modeling of Extreme Values, Springer;</li>
<li>Embrechts, Kluppelberg and Mikosch (1997) Modelling Extreme Events for Insurance and Finance, Springer;</li>
<li>Beirlant, Goegebeur, Teugels and Segers (2004) Statistics of Extremes: Theory and Applications, Wiley;</li>
<li>de Haan and Ferreira (2006) Extreme Value Theory: An Introduction, Springer;</li>
<li>Resnick (1987) Extreme Values, Regular Variation, and Point Processes, Springer;</li>
<li>Leadbetter, Lindgren and Rootzen (1983) Extremes and Related Properties of Random Sequences and Processes, Springer;</li>
<li>Other books:<br>
› – Galambos (1987) The Asymptotic Theory of Extreme Order Statistics, Krieger<br>
› – Gumbel (1958) Statistics of Extremes, Columbia University Press<br>
› – Kotz and Nadarajah (2000) Extreme Value Distributions, Imperial College Press<br>
› – Finkenstadt and Rootzen (2004) Extreme Values in Finance, Telecommunications and the Environment, CRC<br>
› – Reiss and Thomas (2007) Statistical Analysis of Extreme Values, Birkhauser</li>
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<tr><td>Method of evaluation</td><td>30.00% - Scientific review article presentation<br>40.00% - Final exam<br>30.00% - Course Project(s)</td></tr>
<tr><td>Nature of the assignments</td><td>There are three types of assignments:<br>
-Weekly homework: This consists of written theoretical exercises, but will NOT be graded. Solutions will be uploaded on a weekly basis.<br>
-Paper presentations: During the semester, the students will have to present two scientific papers in class; this will count for 30% of the final grade (15% for each presentation).<br>
-Project: Each student will have to do a project, where the extreme-value concepts and methods will be applied in practice to a real dataset using the statistical software R. This project, done individually, will be due near the end of the semester. More details will be given as the semester progresses; this will count for 30% of the final grade.</td></tr>
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## Note

The instructor reserves the right to make changes to this syllabus as necessary.