جامعة الملك عبدالله
للعلوم والتقنية
King Abdullah University of
Science and Technology

Course Syllabus: Numerical Methods/Stochastic Diff Equ. - AMCS 336

Division	Computer, Electrical and Mathematical Sciences & Engineering
Course Number	AMCS 336
Course Title	Numerical Methods/Stochastic Diff Equ.
Academic Semester	Fall
Academic Year	2019/2020
Semester Start Date	08/25/2019
Semester End Date	12/10/2019
Class Schedule (Days & Time)	01:00 PM - 02:30 PM | Mon Wed

Instructor(s)

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Teaching Assistant(s)

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Course Information

Comprehensive Course Description	COURSE OBJECTIVES: The goal of this course is to give basic knowledge of the treatment of stochastic differential equations and their numerical solution, useful for scientific and engineering modeling, guided by some problems in applications in financial mathematics, material science, geophysical flow problems, turbulent diffusion, optimal control theory and Monte Carlo methods. We will discuss basic questions for solving stochastic differential equations, e.g., to determine the price of an option is it more efficient to solve the deterministic Black and Scholes partial differential equation or use a Monte Carlo method based on a stochastic representation? The course treats the basic theory of stochastic differential equations including weak and strong approximation, efficient numerical methods (including multilevel Monte Carlo) and error estimates, the relation between stochastic differential equations and partial differential equations, stochastic partial differential equations, variance reduction, etc. It also addresses Optimal control for ODEs and SDEs and its connections to the Hamilton-Jacobi-Bellman nonlinear PDE.
Course Description from Program Guide	Brownian motion, stochastic integrals and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Weak and strong approximation, efficient numerical methods and error estimates. Jump diffusions.

Goals and Objectives	Upon completion of the course the student should be able to -formulate basic properties of the Wiener process -define the Ito’s integral -apply Ito’s formula for stochastic differentials -discretize a given stochastic differential equation and check resulting approximation properties -implement the Monte Carlo and Multilevel Monte Carlo methods for discretizations of stochastic differential equations -apply variance reduction techniques, including the multilevel Monte Carlo method -state Kolmogorov’s backward (for conditional expectations) and forward equations (for probability density evolution) for a given SDE -control the discretization errors arising in a Monte Carlo method for the weak approximation of SDEs -formulate and discretize Optimal control problems for ODEs and SDEs
Required Knowledge	The prerequisite for the course is knowledge of basic courses in mathematics (probability emphasized here) and numerical analysis, or the equivalent. Some experience of computer programming and the use of UNIX/LINUX/WINDOWS systems or personal computers is assumed.
Reference Texts	The text for the course is the lecture notes: "Stochastic and Partial Differential Equations with Adapted Numerics", authored among others by the teacher. The lecture notes contain ref- erences to additional relevant literature in the subject. We may use additional material as well.
Method of evaluation	**20.00**% - Course Project(s) **50.00**% - Homework /Assignments **30.00**% - Final exam
Nature of the assignments	The homework and computer laboratories constitute a very important part of the course. Computer assignments will be done in MATLAB with existing software for the student to modify and experiment with. Enrolled students will have access to computer facilities with MATLAB at KAUST if needed. There will be an in-class final examination, as scheduled by the registrar during finals week in December.The grading consists of three parts: Homework problems, oral presentations and a written exam. The homework and the presentations are carried out by groups of students. Each group hands in a report for each of the assignments, including the final presentation.
Course Policies	SPECIAL ACCOMMODATIONS If you a have personal activity, a family, or a religious conflict with the course schedule, you may announce it to the instructor. Please contact the instructor by the end of the second week of the term to discuss appropriate accommodations for any conflicts that can be foreseen. For illness-related absences, there are standard procedures to follow.
Additional Information	

Tentative Course Schedule

(Time, topic/emphasis & resources)

Week	Lectures	Topic
1	Mon 08/26/2019 Wed 08/28/2019	Admin details. Motivating examples. Probability refresher. Introduction to Ito Integral. Properties
2	Mon 09/02/2019 Wed 09/04/2019	Ito SDEs as limits of Forward Euler (Euler Maruyama) approximations. Ito SDEs, proof of the existence and uniqueness theorem. Statement of Ito's formula (chain rule). Ito's formula with applications. Stratonovich integrals and SDEs.
3	Mon 09/09/2019 Wed 09/11/2019	Eid Al-Adha Break, Aug. 31– Sept. 10
4	Mon 09/16/2019 Wed 09/18/2019	Systems of Ito SDEs. Regularity of SDE trajectories. Conditional expectation. Kolmogorov Backward equation. Fokker Planck equation. Feynman Kac representation. Applications including estimates for derivatives. Introduction to mathematical finance for european option pricing, derivation of the Black-Scholes PDE equation and its stochastic representation with the Feynman Kac formula. Conditional expectations and non-linear PDEs.
5	Mon 09/23/2019 Wed 09/25/2019	Saudi National Day
6	Mon 09/30/2019 Wed 10/02/2019	Adaptive Monte Carlo method. Variance Reduction for the Monte Carlo method. Control variates, antithetic variates, importance sampling. Variance reduction via conditional expectation. Need of importance sampling to address rare events. Multilevel Monte Carlo.
7	Mon 10/07/2019 Wed 10/09/2019	Introduction to weak approximation of Ordinary Differential Equations (ODEs). Weak approximation of ODEs. Notion of adaptivity and error density. Error representation, local error and dual weight approximation.
8	Mon 10/14/2019 Wed 10/16/2019	Adaptivity and error control for ODEs continued. Analysis of adaptive algorithms (Stopping, Asymptotic Efficiency and Accuracy). SDEs represented by random time changes. Random ODEs. Error representation, a priori estimates for Euler Maruyama weak approximation in SDEs.
9	Mon 10/21/2019 Wed 10/23/2019	Using the first variation to estimate ?. Using a Malliavin weight to estimate greeks. Weak approximation of SDEs by ODEs. Small noise approximation. Weak error estimation and control for Euler Maruyama in SDEs.
10	Mon 10/28/2019 Wed 10/30/2019	Mid-semester break
11	Mon 11/04/2019 Wed 11/06/2019	Mid-semester break
12	Mon 11/11/2019 Wed 11/13/2019	Mid-semester break
13	Mon 11/18/2019 Wed 11/20/2019	Mid-semester break
14	Mon 11/25/2019 Wed 11/27/2019	Mid-semester break
15	Mon 12/02/2019 Wed 12/04/2019	Mid-semester break
16	Mon 12/09/2019	Exams

Note

The instructor reserves the right to make changes to this syllabus as necessary.